COLLISIONS AND TRANSPORT

Temperatures are in eV; the corresponding value of Boltzmann's constant is $k = 1.60 \times 10^{-12}$ erg/eV; masses μ, μ' are in units of the proton mass; $e_\alpha = Z_\alpha e$ is the charge of species α. All other units are cgs except where noted.

Relaxation Rates

Rates are associated with four relaxation processes arising from the interaction of test particles (labeled α) streaming with velocity $\mathbf{v}_\alpha$ through a background of field particles (labeled β):

slowing down	$\dfrac{d\mathbf{v}_\alpha}{dt} = -\nu_s^{\alpha\backslash\beta}\mathbf{v}_\alpha$
transverse diffusion	$\dfrac{d}{dt}(\mathbf{v}_\alpha - \bar{\mathbf{v}}_\alpha)_\perp^2 = \nu_\perp^{\alpha\backslash\beta}{v_\alpha}^2$
parallel diffusion	$\dfrac{d}{dt}(\mathbf{v}_\alpha - \bar{\mathbf{v}}_\alpha)_\parallel^2 = \nu_\parallel^{\alpha\backslash\beta}{v_\alpha}^2$
energy loss	$\dfrac{d}{dt}{v_\alpha}^2 = -\nu_\epsilon^{\alpha\backslash\beta}{v_\alpha}^2,$

where the averages are performed over an ensemble of test particles and a Maxwellian field particle distribution. The exact formulas may be written[19]

$$\begin{aligned}
\nu_s^{\alpha\backslash\beta} &= (1 + m_\alpha/m_\beta)\psi(x^{\alpha\backslash\beta})\nu_0^{\alpha\backslash\beta};\\
\nu_\perp^{\alpha\backslash\beta} &= 2\left[(1 - 1/2x^{\alpha\backslash\beta})\psi(x^{\alpha\backslash\beta}) + \psi'(x^{\alpha\backslash\beta})\right]\nu_0^{\alpha\backslash\beta};\\
\nu_\parallel^{\alpha\backslash\beta} &= \left[\psi(x^{\alpha\backslash\beta})/x^{\alpha\backslash\beta}\right]\nu_0^{\alpha\backslash\beta};\\
\nu_\epsilon^{\alpha\backslash\beta} &= 2\left[(m_\alpha/m_\beta)\psi(x^{\alpha\backslash\beta}) - \psi'(x^{\alpha\backslash\beta})\right]\nu_0^{\alpha\backslash\beta},
\end{aligned}$$

where

$$\nu_0^{\alpha\backslash\beta} = 4\pi {e_\alpha}^2{e_\beta}^2\lambda_{\alpha\beta}n_\beta/{m_\alpha}^2{v_\alpha}^3; \qquad x^{\alpha\backslash\beta} = m_\beta{v_\alpha}^2/2kT_\beta;$$

$$\psi(x) = \frac{2}{\sqrt{\pi}}\int_0^x dt\, t^{1/2}e^{-t}; \quad \psi'(x) = \frac{d\psi}{dx},$$

and $\lambda_{\alpha\beta} = \ln\Lambda_{\alpha\beta}$ is the Coulomb logarithm (see below). Limiting forms of ν_s, $\nu_\perp$ and $\nu_\parallel$ are given in the following table. All the expressions shown have units $\mathrm{cm}^3\,\mathrm{sec}^{-1}$. Test particle energy ϵ and field particle temperature T are both in eV; $\mu = m_i/m_p$ where m_p is the proton mass; Z is ion charge state; in electron–electron and ion–ion encounters, field particle quantities are distinguished by a prime. The two expressions given below for each rate hold for very slow ($x^{\alpha\backslash\beta} \ll 1$) and very fast ($x^{\alpha\backslash\beta} \gg 1$) test particles, respectively.

	Slow	Fast
Electron–electron		
$\nu_s^{e\backslash e'}/n_{e'}\lambda_{ee'}$	$\approx 5.8\times 10^{-6}T^{-3/2}$	$\longrightarrow 7.7\times 10^{-6}\epsilon^{-3/2}$
$\nu_\perp^{e\backslash e'}/n_{e'}\lambda_{ee'}$	$\approx 5.8\times 10^{-6}T^{-1/2}\epsilon^{-1}$	$\longrightarrow 7.7\times 10^{-6}\epsilon^{-3/2}$
$\nu_\parallel^{e\backslash e'}/n_{e'}\lambda_{ee'}$	$\approx 2.9\times 10^{-6}T^{-1/2}\epsilon^{-1}$	$\longrightarrow 3.9\times 10^{-6}T\epsilon^{-5/2}$
Electron–ion		
$\nu_s^{e\backslash i}/n_iZ^2\lambda_{ei}$	$\approx 0.23\mu^{3/2}T^{-3/2}$	$\longrightarrow 3.9\times 10^{-6}\epsilon^{-3/2}$
$\nu_\perp^{e\backslash i}/n_iZ^2\lambda_{ei}$	$\approx 2.5\times 10^{-4}\mu^{1/2}T^{-1/2}\epsilon^{-1}$	$\longrightarrow 7.7\times 10^{-6}\epsilon^{-3/2}$
$\nu_\parallel^{e\backslash i}/n_iZ^2\lambda_{ei}$	$\approx 1.2\times 10^{-4}\mu^{1/2}T^{-1/2}\epsilon^{-1}$	$\longrightarrow 2.1\times 10^{-9}\mu^{-1}T\epsilon^{-5/2}$
Ion–electron		
$\nu_s^{i\backslash e}/n_eZ^2\lambda_{ie}$	$\approx 1.6\times 10^{-9}\mu^{-1}T^{-3/2}$	$\longrightarrow 1.7\times 10^{-4}\mu^{1/2}\epsilon^{-3/2}$
$\nu_\perp^{i\backslash e}/n_eZ^2\lambda_{ie}$	$\approx 3.2\times 10^{-9}\mu^{-1}T^{-1/2}\epsilon^{-1}$	$\longrightarrow 1.8\times 10^{-7}\mu^{-1/2}\epsilon^{-3/2}$
$\nu_\parallel^{i\backslash e}/n_eZ^2\lambda_{ie}$	$\approx 1.6\times 10^{-9}\mu^{-1}T^{-1/2}\epsilon^{-1}$	$\longrightarrow 1.7\times 10^{-4}\mu^{1/2}T\epsilon^{-5/2}$

Ion–ion

$$\frac{\nu_s^{i\backslash i'}}{n_{i'}Z^2Z'^2\lambda_{ii'}} \approx 6.8\times 10^{-8}\frac{\mu'^{1/2}}{\mu}\left(1+\frac{\mu'}{\mu}\right)^{-1/2}T^{-3/2}$$
$$\longrightarrow 9.0\times 10^{-8}\left(\frac{1}{\mu}+\frac{1}{\mu'}\right)\frac{\mu^{1/2}}{\epsilon^{3/2}}$$

$$\frac{\nu_\perp^{i\backslash i'}}{n_{i'}Z^2Z'^2\lambda_{ii'}} \approx 1.4\times 10^{-7}\mu'^{1/2}\mu^{-1}T^{-1/2}\epsilon^{-1}$$
$$\longrightarrow 1.8\times 10^{-7}\mu^{-1/2}\epsilon^{-3/2}$$

$$\frac{\nu_\parallel^{i\backslash i'}}{n_{i'}Z^2Z'^2\lambda_{ii'}} \approx 6.8\times 10^{-8}\mu'^{1/2}\mu^{-1}T^{-1/2}\epsilon^{-1}$$
$$\longrightarrow 9.0\times 10^{-8}\mu^{1/2}\mu'^{-1}T\epsilon^{-5/2}$$

In the same limits, the energy transfer rate follows from the identity

$$\nu_\epsilon = 2\nu_s - \nu_\perp - \nu_\parallel,$$

except for the case of fast electrons or fast ions scattered by ions, where the leading terms cancel. Then the appropriate forms are

$$\nu_\epsilon^{e\backslash i} \longrightarrow 4.2\times 10^{-9}n_iZ^2\lambda_{ei}$$
$$\left[\epsilon^{-3/2}\mu^{-1} - 8.9\times 10^4(\mu/T)^{1/2}\epsilon^{-1}\exp(-1836\mu\epsilon/T)\right]\ \mathrm{sec}^{-1}$$

and

$$\nu_\epsilon^{i\backslash i'} \longrightarrow 1.8 \times 10^{-7} n_{i'} Z^2 Z'^2 \lambda_{ii'}$$
$$\left[\epsilon^{-3/2}\mu^{1/2}/\mu' - 1.1(\mu'/T)^{1/2}\epsilon^{-1}\exp(-\mu'\epsilon/T)\right] \sec^{-1}.$$

In general, the energy transfer rate $\nu_\epsilon^{\alpha\backslash\beta}$ is positive for $\epsilon > \epsilon_\alpha{}^*$ and negative for $\epsilon < \epsilon_\alpha{}^*$, where $x^* = (m_\beta \backslash m_\alpha)\epsilon_\alpha{}^*/T_\beta$ is the solution of $\psi'(x^*) = (m_\alpha \backslash m_\beta)\psi(x^*)$. The ratio $\epsilon_\alpha{}^*/T_\beta$ is given for a number of specific α, β in the following table:

$\alpha\backslash\beta$	$i\backslash e$	$e\backslash e$, $i\backslash i$	$e\backslash p$	$e\backslash$D	$e\backslash$T, $e\backslash$He^3	$e\backslash$He^4
$\dfrac{\epsilon_\alpha{}^*}{T_\beta}$	1.5	0.98	4.8×10^{-3}	2.6×10^{-3}	1.8×10^{-3}	1.4×10^{-3}

When both species are near Maxwellian, with $T_i \lesssim T_e$, there are just two characteristic collision rates. For $Z = 1$,

$$\nu_e = 2.9 \times 10^{-6} n\lambda T_e{}^{-3/2} \sec^{-1};$$
$$\nu_i = 4.8 \times 10^{-8} n\lambda T_i{}^{-3/2}\mu^{-1/2} \sec^{-1}.$$

Temperature Isotropization

Isotropization is described by

$$\frac{dT_\perp}{dt} = -\frac{1}{2}\frac{dT_\parallel}{dt} = -\nu_T^\alpha (T_\perp - T_\parallel),$$

where, if $A \equiv T_\perp/T_\parallel - 1 > 0$,

$$\nu_T^\alpha = \frac{2\sqrt{\pi} e_\alpha{}^2 e_\beta{}^2 n_\alpha \lambda_{\alpha\beta}}{m_\alpha{}^{1/2}(kT_\parallel)^{3/2}} A^{-2}\left[-3 + (A+3)\frac{\tan^{-1}(A^{1/2})}{A^{1/2}}\right].$$

If $A < 0$, $\tan^{-1}(A^{1/2})/A^{1/2}$ is replaced by $\tanh^{-1}(-A)^{1/2}/(-A)^{1/2}$. For $T_\perp \approx T_\parallel \equiv T$,

$$\nu_T^e = 8.2 \times 10^{-7} n\lambda T^{-3/2} \sec^{-1};$$
$$\nu_T^i = 1.9 \times 10^{-8} n\lambda Z^2 \mu^{-1/2} T^{-3/2} \sec^{-1}.$$

Thermal Equilibration

If the components of a plasma have different temperatures, but no relative drift, equilibration is described by

$$\frac{dT_\alpha}{dt} = \sum_\beta \bar{\nu}_\epsilon^{\alpha\backslash\beta}(T_\beta - T_\alpha),$$

where

$$\bar{\nu}_\epsilon^{\alpha\backslash\beta} = 1.8 \times 10^{-19} \frac{(m_\alpha m_\beta)^{1/2} {Z_\alpha}^2 {Z_\beta}^2 n_\beta \lambda_{\alpha\beta}}{(m_\alpha T_\beta + m_\beta T_\alpha)^{3/2}} \,\mathrm{sec}^{-1}.$$

For electrons and ions with $T_e \approx T_i \equiv T$, this implies

$$\bar{\nu}_\epsilon^{e\backslash i}/n_i = \bar{\nu}_\epsilon^{i\backslash e}/n_e = 3.2 \times 10^{-9} Z^2 \lambda/\mu T^{3\backslash 2} \mathrm{cm}^3\,\mathrm{sec}^{-1}.$$

Coulomb Logarithm

For test particles of mass m_α and charge $e_\alpha = Z_\alpha e$ scattering off field particles of mass m_β and charge $e_\beta = Z_\beta e$, the Coulomb logarithm is defined as $\lambda = \ln\Lambda \equiv \ln(r_{\max}/r_{\min})$. Here $r_{\min}$ is the larger of $e_\alpha e_\beta/m_{\alpha\beta}\bar{u}^2$ and $\hbar/2m_{\alpha\beta}\bar{u}$, averaged over both particle velocity distributions, where $m_{\alpha\beta} = m_\alpha m_\beta/(m_\alpha + m_\beta)$ and $\mathbf{u} = \mathbf{v}_\alpha - \mathbf{v}_\beta$; $r_{\max} = (4\pi \sum n_\gamma {e_\gamma}^2/kT_\gamma)^{-1/2}$, where the summation extends over all species γ for which $\bar{u}^2 < {v_{T\gamma}}^2 = kT_\gamma/m_\gamma$. If this inequality cannot be satisfied, or if either $\bar{u}{\omega_{c\alpha}}^{-1} < r_{\max}$ or $\bar{u}{\omega_{c\beta}}^{-1} < r_{\max}$, the theory breaks down. Typically $\lambda \approx$ 10–20. Corrections to the transport coefficients are $O(\lambda^{-1})$; hence the theory is good only to $\sim 10\%$ and fails when $\lambda \sim 1$.

The following cases are of particular interest:

(a) Thermal electron–electron collisions

$$\begin{aligned} \lambda_{ee} &= 23 - \ln({n_e}^{1/2}{T_e}^{-3/2}), && T_e \lesssim 10\,\mathrm{eV}; \\ &= 24 - \ln({n_e}^{1/2}{T_e}^{-1}), && T_e \gtrsim 10\,\mathrm{eV}. \end{aligned}$$

(b) Electron–ion collisions

$$\begin{aligned} \lambda_{ei} = \lambda_{ie} &= 23 - \ln\left({n_e}^{1/2} Z T_e^{-3/2}\right), && T_i m_e/m_i < T_e < 10Z^2\,\mathrm{eV}; \\ &= 24 - \ln\left({n_e}^{1/2} T_e^{-1}\right), && T_i m_e/m_i < 10Z^2\,\mathrm{eV} < T_e \\ &= 30 - \ln\left({n_i}^{1/2} {T_i}^{-3/2} Z^2 \mu^{-1}\right), && T_e < T_i Z m_e/m_i. \end{aligned}$$

(c) Mixed ion–ion collisions

$$\lambda_{ii'} = \lambda_{i'i} = 23 - \ln\left[\frac{ZZ'(\mu+\mu')}{\mu T_{i'} + \mu' T_i}\left(\frac{n_i Z^2}{T_i} + \frac{n_{i'} Z'^2}{T_{i'}}\right)^{1/2}\right].$$

(d) Counterstreaming ions (relative velocity $v_D = \beta_D c$) in the presence of warm electrons, $kT_i/m_i, kT_{i'}/m_{i'} < v_D{}^2 < kT_e/m_e$

$$\lambda_{ii'} = \lambda_{i'i} = 35 - \ln\left[\frac{ZZ'(\mu+\mu')}{\mu\mu'\beta_D{}^2}\left(\frac{n_e}{T_e}\right)^{1/2}\right].$$

Fokker-Planck Equation

$$\frac{Df^\alpha}{Dt} \equiv \frac{\partial f^\alpha}{\partial t} + \mathbf{v}\cdot\nabla f^\alpha + \mathbf{F}\cdot\nabla_{\mathbf{v}} f^\alpha = \left(\frac{\partial f^\alpha}{\partial t}\right)_{\text{coll}},$$

where $\mathbf{F}$ is an external force field. The general form of the collision integral is $(\partial f^\alpha/\partial t)_{\text{coll}} = -\sum_\beta \nabla_{\mathbf{v}}\cdot\mathbf{J}^{\alpha\backslash\beta}$, with

$$\mathbf{J}^{\alpha\backslash\beta} = 2\pi\lambda_{\alpha\beta}\frac{e_\alpha{}^2 e_\beta{}^2}{m_\alpha}\int d^3v'(u^2\mathsf{I} - \mathbf{u}\mathbf{u})u^{-3} \cdot\left\{\frac{1}{m_\beta}f^\alpha(\mathbf{v})\nabla_{\mathbf{v}'}f^\beta(\mathbf{v}') - \frac{1}{m_\alpha}f^\beta(\mathbf{v}')\nabla_{\mathbf{v}}f^\alpha(\mathbf{v})\right\}$$

(Landau form) where $\mathbf{u} = \mathbf{v}' - \mathbf{v}$ and I is the unit dyad, or alternatively,

$$\mathbf{J}^{\alpha\backslash\beta} = 4\pi\lambda_{\alpha\beta}\frac{e_\alpha{}^2 e_\beta{}^2}{m_\alpha{}^2}\left\{f^\alpha(\mathbf{v})\nabla_{\mathbf{v}}H(\mathbf{v}) - \frac{1}{2}\nabla_{\mathbf{v}}\cdot\left[f^\alpha(\mathbf{v})\nabla_{\mathbf{v}}\nabla_{\mathbf{v}}G(\mathbf{v})\right]\right\},$$

where the Rosenbluth potentials are

$$G(\mathbf{v}) = \int f^\beta(\mathbf{v}')u\,d^3v'$$

$$H(\mathbf{v}) = \left(1 + \frac{m_\alpha}{m_\beta}\right)\int f^\beta(\mathbf{v}')u^{-1}d^3v'.$$

If species α is a weak beam (number and energy density small compared with background) streaming through a Maxwellian plasma, then

$$\mathbf{J}^{\alpha\backslash\beta} = -\frac{m_\alpha}{m_\alpha + m_\beta}\nu_s^{\alpha\backslash\beta}\mathbf{v}f^\alpha - \frac{1}{2}\nu_\parallel^{\alpha\backslash\beta}\mathbf{v}\mathbf{v}\cdot\nabla_{\mathbf{v}}f^\alpha$$
$$-\frac{1}{4}\nu_\perp^{\alpha\backslash\beta}\left(v^2\mathsf{I} - \mathbf{v}\mathbf{v}\right)\cdot\nabla_{\mathbf{v}}f^\alpha.$$

B-G-K Collision Operator

For distribution functions with no large gradients in velocity space, the Fokker-Planck collision terms can be approximated according to

$$\frac{Df_e}{Dt} = \nu_{ee}(F_e - f_e) + \nu_{ei}(\bar{F}_e - f_e);$$

$$\frac{Df_i}{Dt} = \nu_{ie}(\bar{F}_i - f_i) + \nu_{ii}(F_i - f_i).$$

The respective slowing-down rates $\nu_s^{\alpha\backslash\beta}$ given in the Relaxation Rate section above can be used for $\nu_{\alpha\beta}$, assuming slow ions and fast electrons, with ϵ replaced by T_α. (For ν_{ee} and ν_{ii}, one can equally well use $\nu_\perp$, and the result is insensitive to whether the slow- or fast-test-particle limit is employed.) The Maxwellians F_α and $\bar{F}_\alpha$ are given by

$$F_\alpha = n_\alpha\left(\frac{m_\alpha}{2\pi kT_\alpha}\right)^{3/2}\exp\left\{-\left[\frac{m_\alpha(\mathbf{v}-\mathbf{v}_\alpha)^2}{2kT_\alpha}\right]\right\};$$

$$\bar{F}_\alpha = n_\alpha\left(\frac{m_\alpha}{2\pi k\bar{T}_\alpha}\right)^{3/2}\exp\left\{-\left[\frac{m_\alpha(\mathbf{v}-\bar{\mathbf{v}}_\alpha)^2}{2k\bar{T}_\alpha}\right]\right\},$$

where n_α, $\mathbf{v}_\alpha$ and T_α are the number density, mean drift velocity, and effective temperature obtained by taking moments of f_α. Some latitude in the definition of $\bar{T}_\alpha$ and $\bar{\mathbf{v}}_\alpha$ is possible;[20] one choice is $\bar{T}_e = T_i$, $\bar{T}_i = T_e$, $\bar{\mathbf{v}}_e = \mathbf{v}_i$, $\bar{\mathbf{v}}_i = \mathbf{v}_e$.

Transport Coefficients

Transport equations for a multispecies plasma:

$$\frac{d^\alpha n_\alpha}{dt} + n_\alpha\nabla\cdot\mathbf{v}_\alpha = 0;$$

$$m_\alpha n_\alpha\frac{d^\alpha\mathbf{v}_\alpha}{dt} = -\nabla p_\alpha - \nabla\cdot\mathsf{P}_\alpha + Z_\alpha e n_\alpha\left[\mathbf{E} + \frac{1}{c}\mathbf{v}_\alpha\times\mathbf{B}\right] + \mathbf{R}_\alpha;$$

$$\frac{3}{2} n_\alpha \frac{d^\alpha kT_\alpha}{dt} + p_\alpha \nabla \cdot \mathbf{v}_\alpha = -\nabla \cdot \mathbf{q}_\alpha - P_\alpha : \nabla \mathbf{v}_\alpha + Q_\alpha .$$

Here $d^\alpha/dt \equiv \partial/\partial t + \mathbf{v}_\alpha \cdot \nabla$; $p_\alpha = n_\alpha kT_\alpha$, where k is Boltzmann's constant; $\mathbf{R}_\alpha = \sum_\beta \mathbf{R}_{\alpha\beta}$ and $Q_\alpha = \sum_\beta Q_{\alpha\beta}$, where $\mathbf{R}_{\alpha\beta}$ and $Q_{\alpha\beta}$ are respectively the momentum and energy gained by the αth species through collisions with the βth; P_α is the stress tensor; and $\mathbf{q}_\alpha$ is the heat flow.

The transport coefficients in a simple two-component plasma (electrons and singly charged ions) are tabulated below. Here $\|$ and $\perp$ refer to the direction of the magnetic field $\mathbf{B} = \mathbf{b}B$; $\mathbf{u} = \mathbf{v}_e - \mathbf{v}_i$ is the relative streaming velocity; $n_e = n_i \equiv n$; $\mathbf{j} = -ne\mathbf{u}$ is the current; $\omega_{ce} = 1.76 \times 10^7 B \sec^{-1}$ and $\omega_{ci} = (m_e/m_i)\omega_{ce}$ are the electron and ion gyrofrequencies, respectively; and the basic collisional times are taken to be

$$\tau_e = \frac{3\sqrt{m_e}(kT_e)^{3/2}}{4\sqrt{2\pi}\, n\lambda e^4} = 3.44 \times 10^5 \frac{{T_e}^{3/2}}{n\lambda} \sec,$$

where λ is the Coulomb logarithm, and

$$\tau_i = \frac{3\sqrt{m_i}(kT_i)^{3/2}}{4\sqrt{\pi} n\, \lambda e^4} = 2.09 \times 10^7 \frac{{T_i}^{3/2}}{n\lambda} \mu^{1/2} \sec.$$

In the limit of large fields ($\omega_{c\alpha}\tau_\alpha \gg 1$, $\alpha = i, e$) the transport processes may be summarized as follows:[21]

momentum transfer	$\mathbf{R}_{ei} = -\mathbf{R}_{ie} \equiv \mathbf{R} = \mathbf{R}_\mathbf{u} + \mathbf{R}_T$;
frictional force	$\mathbf{R}_\mathbf{u} = ne(\mathbf{j}_\| / \sigma_\| + \mathbf{j}_\perp / \sigma_\perp)$;
electrical conductivities	$\sigma_\| = 1.96\sigma_\perp$; $\sigma_\perp = ne^2\tau_e/m_e$;
thermal force	$\mathbf{R}_T = -0.71 n \nabla_\| (kT_e) - \dfrac{3n}{2\omega_{ce}\tau_e} \mathbf{b} \times \nabla_\perp (kT_e)$;
ion heating	$Q_i = \dfrac{3m_e}{m_i} \dfrac{nk}{\tau_e}(T_e - T_i)$;
electron heating	$Q_e = -Q_i - \mathbf{R} \cdot \mathbf{u}$;
ion heat flux	$\mathbf{q}_i = -\kappa^i_\| \nabla_\| (kT_i) - \kappa^i_\perp \nabla_\perp (kT_i) + \kappa^i_\wedge \mathbf{b} \times \nabla_\perp (kT_i)$;
ion thermal conductivities	$\kappa^i_\| = 3.9 \dfrac{nkT_i\tau_i}{m_i}$; $\kappa^i_\perp = \dfrac{2nkT_i}{m_i {\omega_{ci}}^2 \tau_i}$; $\kappa^i_\wedge = \dfrac{5nkT_i}{2m_i\omega_{ci}}$;
electron heat flux	$\mathbf{q}_e = \mathbf{q}^e_\mathbf{u} + \mathbf{q}^e_T$;
frictional heat flux	$\mathbf{q}^e_\mathbf{u} = 0.71 nkT_e \mathbf{u}_\| + \dfrac{3nkT_e}{2\omega_{ce}\tau_e} \mathbf{b} \times \mathbf{u}_\perp$;

thermal gradient heat flux	$\mathbf{q}_T^e = -\kappa_\parallel^e \nabla_\parallel(kT_e) - \kappa_\perp^e \nabla_\perp(kT_e) - \kappa_\wedge^e \mathbf{b} \times \nabla_\perp(kT_e);$
electron thermal conductivities	$\kappa_\parallel^e = 3.2 \dfrac{nkT_e\tau_e}{m_e}; \quad \kappa_\perp^e = 4.7 \dfrac{nkT_e}{m_e {\omega_{ce}}^2 \tau_e}; \quad \kappa_\wedge^e = \dfrac{5nkT_e}{2m_e\omega_{ce}};$
stress tensor (either species)	$P_{xx} = -\dfrac{\eta_0}{2}(W_{xx} + W_{yy}) - \dfrac{\eta_1}{2}(W_{xx} - W_{yy}) - \eta_3 W_{xy};$
	$P_{yy} = -\dfrac{\eta_0}{2}(W_{xx} + W_{yy}) + \dfrac{\eta_1}{2}(W_{xx} - W_{yy}) + \eta_3 W_{xy};$
	$P_{xy} = P_{yx} = -\eta_1 W_{xy} + \dfrac{\eta_3}{2}(W_{xx} - W_{yy});$
	$P_{xz} = P_{zx} = -\eta_2 W_{xz} - \eta_4 W_{yz};$
	$P_{yz} = P_{zy} = -\eta_2 W_{yz} + \eta_4 W_{xz};$
	$P_{zz} = -\eta_0 W_{zz}$

(here the z axis is defined parallel to $\mathbf{B}$);

ion viscosity	$\eta_0^i = 0.96 nkT_i\tau_i; \quad \eta_1^i = \dfrac{3nkT_i}{10{\omega_{ci}}^2\tau_i}; \quad \eta_2^i = \dfrac{6nkT_i}{5{\omega_{ci}}^2\tau_i};$
	$\eta_3^i = \dfrac{nkT_i}{2\omega_{ci}}; \quad \eta_4^i = \dfrac{nkT_i}{\omega_{ci}};$
electron viscosity	$\eta_0^e = 0.73 nkT_e\tau_e; \quad \eta_1^e = 0.51 \dfrac{nkT_e}{{\omega_{ce}}^2\tau_e}; \quad \eta_2^e = 2.0 \dfrac{nkT_e}{{\omega_{ce}}^2\tau_e};$
	$\eta_3^e = -\dfrac{nkT_e}{2\omega_{ce}}; \quad \eta_4^e = -\dfrac{nkT_e}{\omega_{ce}}.$

For both species the rate-of-strain tensor is defined as

$$W_{jk} = \frac{\partial v_j}{\partial x_k} + \frac{\partial v_k}{\partial x_j} - \frac{2}{3}\delta_{jk}\nabla \cdot \mathbf{v}.$$

When $\mathbf{B} = 0$ the following simplifications occur:

$$\mathbf{R}_\mathbf{u} = ne\mathbf{j}/\sigma_\parallel; \quad \mathbf{R}_T = -0.71 n\nabla(kT_e); \quad \mathbf{q}_i = -\kappa_\parallel^i \nabla(kT_i);$$

$$\mathbf{q}_\mathbf{u}^e = 0.71 nkT_e\mathbf{u}; \quad \mathbf{q}_T^e = -\kappa_\parallel^e \nabla(kT_e); \quad P_{jk} = -\eta_0 W_{jk}.$$

For $\omega_{ce}\tau_e \gg 1 \gg \omega_{ci}\tau_i$, the electrons obey the high-field expressions and the ions obey the zero-field expressions.

Collisional transport theory is applicable when (1) macroscopic time rates of change satisfy $d/dt \ll 1/\tau$, where τ is the longest collisional time scale, and (in the absence of a magnetic field) (2) macroscopic length scales L satisfy $L \gg l$, where $l = \bar{v}\tau$ is the mean free path. In a strong field, $\omega_{ce}\tau \gg 1$, condition (2) is replaced by $L_\parallel \gg l$ and $L_\perp \gg \sqrt{lr_e}$ ($L_\perp \gg r_e$ in a uniform field), where $L_{||}$ is a macroscopic scale parallel to the field $\mathbf{B}$ and $L_\perp$ is the smaller of $B/|\nabla_\perp B|$ and the transverse plasma dimension. In addition, the standard transport coefficients are valid only when (3) the Coulomb logarithm satisfies $\lambda \gg 1$; (4) the electron gyroradius satisfies $r_e \gg \lambda_D$, or $8\pi n_e m_e c^2 \gg B^2$; (5) relative drifts $\mathbf{u} = \mathbf{v}_\alpha - \mathbf{v}_\beta$ between two species are small compared with the thermal velocities, i.e., $u^2 \ll kT_\alpha/m_\alpha$, kT_β/m_β; and (6) anomalous transport processes owing to microinstabilities are negligible.

Weakly Ionized Plasmas

Collision frequency for scattering of charged particles of species α by neutrals is

$$\nu_\alpha = n_0 \sigma_s^{\alpha\backslash 0}(kT_\alpha/m_\alpha)^{1/2},$$

where n_0 is the neutral density and $\sigma_s^{\alpha\backslash 0}$ is the cross section, typically $\sim 5 \times 10^{-15}$ cm^2 and weakly dependent on temperature.

When the system is small compared with a Debye length, $L \ll \lambda_D$, the charged particle diffusion coefficients are

$$D_\alpha = kT_\alpha/m_\alpha\nu_\alpha,$$

In the opposite limit, both species diffuse at the ambipolar rate

$$D_A = \frac{\mu_i D_e - \mu_e D_i}{\mu_i - \mu_e} = \frac{(T_i + T_e)D_i D_e}{T_i D_e + T_e D_i},$$

where $\mu_\alpha = e_\alpha/m_\alpha\nu_\alpha$ is the mobility. The conductivity σ_α satisfies $\sigma_\alpha = n_\alpha e_\alpha \mu_\alpha$.

In the presence of a magnetic field $\mathbf{B}$ the scalars μ and σ become tensors,

$$\mathbf{J}^\alpha = \boldsymbol{\sigma}^\alpha \cdot \mathbf{E} = \sigma_{||}^\alpha \mathbf{E}_{||} + \sigma_\perp^\alpha \mathbf{E}_\perp + \sigma_\wedge^\alpha \mathbf{E} \times \mathbf{b},$$

where $\mathbf{b} = \mathbf{B}/B$ and

$$\sigma_{||}^\alpha = n_\alpha {e_\alpha}^2/m_\alpha\nu_\alpha;$$

$$\sigma_\perp^\alpha = \sigma_{||}^\alpha {\nu_\alpha}^2/({\nu_\alpha}^2 + \omega_{c\alpha}^2);$$

$$\sigma_\wedge^\alpha = \sigma_{||}^\alpha \nu_\alpha\omega_{c\alpha}/({\nu_\alpha}^2 + \omega_{c\alpha}^2).$$

Here $\sigma_\perp$ and $\sigma_\wedge$ are the Pedersen and Hall conductivities, respectively.